



College and Career Readiness Standards-in-Action

Foundational
Unit

4

FACILITATOR GUIDE
MATHEMATICS

CONNECTING STANDARDS FOR MATHEMATICAL PRACTICE TO CONTENT

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Background and Purpose

The College and Career Readiness (CCR) Standards for Adult Education for mathematics have two central parts: the Standards for Mathematical Practice and the mathematical content standards. The Standards for Mathematical Practice describe habits of mind that mathematics educators should cultivate in their students.

These Standards for Mathematical Practice rest on “processes and proficiencies” with established significance in mathematics education. They include skills such as complex problem-solving, reasoning, modeling, strategic use of appropriate tools, precise communication, and making use of patterns and structure. The mathematical content standards are a sequence of topics and performances that reflect a balanced combination of procedural fluency, conceptual understanding, and application. This content is intended to be connected to the Standards for Mathematical Practice across domains and at each level of learning.

Other sets of nationally recognized standards have long acknowledged the importance of the processes and proficiencies now articulated in the CCR Standards for Mathematical Practice. The National Council of Teachers of Mathematics identified the process standards of problem-solving, reasoning and proof, communication, representation, and connections.¹ Similarly, strands of mathematical proficiency were specified by the National Research Council. They include adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition.²

¹ National Council of Teachers of Mathematics. *Curriculum and Evaluation Standards for School Mathematics* (1989) and *Principles and Standards for School Mathematics* (2000).

² Adding it Up: Helping Children Learn Mathematics. National Research Council, Mathematics Learning Study Committee, 2001.

Surveys done with employers and professors of entry-level, credit-bearing college mathematics courses indicate that students need the processes and proficiencies described in the Standards for Mathematical Practice.³ They indicate that it is not enough for students to be able to perform mathematical operations and procedures. Students who are college- and career-ready need to be able to make sense of problems and determine how mathematics can be used to solve them. Students need to be able to justify their own reasoning, and critique the reasoning of others. They need to be able to model problems that occur in everyday life, society, and the workplace.

The Standards for Mathematical Practice define ways in which students need to be able to engage with the subject matter as they grow in mathematical maturity and expertise across levels. Including the Standards for Mathematical Practice in lessons provides students a flexible base from which to solve similar problems, represent problems coherently, justify conclusions with sound reasoning, and use technology and tools mindfully.

This unit provides participants an opportunity to delve deeply into the Standards for Mathematical Practice and to think about ways in which connections can be made between the Standards for Mathematical Practice and the content of the CCR Standards for Adult Education. In particular, those CCR content standards that set the expectation for “understanding” provide great opportunity for forging these connections and for identifying ways that the Standards for Mathematical Practice can be used to reinforce those areas that merit focus. In this unit, participants will focus on how to integrate the Standards for Mathematical Practice into lessons related to specific content to enrich student learning.

³ *Ready or Not: Creating a High School Diploma That Counts*. American Diploma Project, 2004; 2009 ACT National Curriculum Survey.

Overview

To begin, participants will independently conduct a close read of the Standards for Mathematical Practice and highlight keywords and phrases. This will allow participants who are not familiar with the Standards for Mathematical Practice to get a better grasp of what they are. It also will provide participants who are familiar with the Standards for Mathematical Practice an opportunity to reacquaint themselves with their substance.

The first part of this unit (Matching the Standards for Mathematical Practice to Content Standards) asks participants to read and analyze the requirements of a Level B CCR Standard, and imagine a lesson that might target that standard. Then, participants will determine which Standards for Mathematical Practice would be central to that lesson and which might be used in a supporting role. Participants also will be asked to identify which Standards for Mathematical Practice are not relevant to the imagined lesson or the standard it targets. All three observations are important in understanding how to connect the Standards for Mathematical Practice to mathematical content.

Part Two of this unit (Enriching a Mathematics Lesson) will ask participants to read and analyze a sample lesson and look for opportunities for students to engage the specific Standards for Mathematical Practice. Participants will look for those Standards for Mathematical Practice that are central to the lesson, those Standards for Mathematical Practice that support the Standards for Mathematical Practice central to the lesson, and those that are not relevant to the lesson.

Materials You Need

For Participants – Part One: Matching Standards for Mathematical Practice to Content Standards (one copy per participant):

- Directions for Participants
- Worksheet: Matching Standards for Mathematical Practice to Content Standards
- Resource: CCR Standards for Mathematical Practice
- Resource: CCR Standards for Adult Education (one copy per table)

For Participants – Part Two: Enriching a Mathematics Lesson (one copy per participant):

- Directions for Participants
- Worksheet: Enriching a Mathematics Lesson
- Resource: Math Lesson—Equivalent Fractions⁴ (make multiple copies for each table)
- Resource: CCR Standards for Mathematical Practice
- Resource: CCR Standards for Adult Education (one copy per table)

For Facilitator:

- Unit 4 PowerPoint Presentation: Connecting Standards for Mathematical Practice to Content
- Answer Key/Rationales for Part One: Matching Standards for Mathematical Practice to Content Standard
- Answer Key/Rationales for Part Two: Enriching a Mathematics Lesson

⁴ For the purposes of this activity, the lesson from the New York State Education Department Common Core curriculum (EngageNY.org) has been extracted from a complete mathematics module and modified slightly.

Time Frame to Complete the Unit

Allow **120 minutes** to complete this unit, using the following guidance to help divide the time:

- 20 minutes – Introduce the unit.
- 20 minutes – Complete the Matching Standards for Mathematical Practice to Content Standards activity.
- 60 minutes – Complete the Enriching a Mathematics Lesson activity.
- 20 minutes – Discuss reflections.

NOTE: Participants who are unfamiliar with the Standards for Mathematical Practice will need more time to understand their meaning to complete the activity.

Guidelines for Implementation

Step 1: Preparations for Part One and Part Two, Matching the Standards for Mathematical Practice to Content Standards and Enriching a Mathematics Lesson

- a) Create small groups of participants, ideally with four to eight participants at each table.** The maximum size of a group for this session depends on your space, need, and comfort level. A guiding principle is to make sure the group is small enough that you can be in touch with each table of participants to determine whether they understand the concepts and are fully engaged or they are struggling and need more support.
- b) For best results, select table leaders in advance or ask each table of participants during the session to choose one person to be their lead.** The table leader will be responsible for keeping track of time, bringing participants together at the appropriate times,

making sure participants are moving along, sharing information at appropriate times from the answer keys and rationales, and notifying you if there are questions or the group needs more support. (If table leaders are selected in advance they should be given copies of the PowerPoint Presentation, materials, and answer key to prepare for the session.)

- c) **As a general strategy, be prepared to circulate around the room when participants are working individually, or in pairs.** Circulating will allow you to check on their understanding and be readily available to answer questions.
- d) **Prepare the materials for Parts One and Two for participants.** Copy the resource CCR Standards for Mathematical Practice on colored paper so that participants can spot it easily. Make multiple copies of the Equivalent Fractions lesson, although one per person may not be necessary. Provide a single copy of the CCR Standards for each table. In advance of the session, advise participants who think they might need their own copy of the CCR Standards to bring it with them.
- e) **To increase participation and understanding, send copies of the resource CCR Standards for Mathematical Practice to all participants in advance.** Ask them to read each Standard of Mathematical Practice and highlight words and phrases that best exemplify the meaning and requirements of each. You should provide participants time to reacquaint themselves with the Standards for Mathematical Practice at the training.
- f) **Become familiar with the PowerPoint Presentation and materials, including the answer keys.** This will allow you to be at ease with the information and flow of the unit. Detailed notes are provided in the

PowerPoint Presentation to help you prepare for the session. In particular, notes for each slide include the identification of the Big Idea, Facilitator Talking Points, and Facilitator Notes. These can help you frame your presentation and provide you important detail and context. This information is coupled with that offered in this Facilitator Guide—including the research base, rationale, advice, and other guidelines—to give you the support and guidance you require.

The answer key is for your edification and is not meant to be handed out to participants. It includes “right” answers, but they are not necessarily the only right answers; it includes well-supported judgments that will guide you as you reflect on participant questions and answers.

Step 2: Implementation of Part One, Matching Standards for Mathematical Practice to Content Standards (20 minutes to introduce; 20 additional minutes working in pairs or small groups)

Introduce the three key advances in the CCR Standards for Mathematics.

Slide 2: Discuss with participants the three key advances and how they interact with one another and build toward college and career readiness. It is important to understand that the CCR Standards were developed to reflect and exemplify the three key advances of *focus*, *coherence*, and *rigor*.

Remind participants that in Unit 1 they learned about *focus* and used the Major Work of the Levels resource to become familiar with the topics emphasized within each CCR level. In Unit 2, they learned about *coherence* and how the CCR Standards are designed with a flow and progression of content within and across levels, grounded in the major

work of the levels. In Unit 3 they learned about recognizing and applying the three components of *rigor*: 1) conceptual understanding; 2) procedural skill and fluency; and 3) application of mathematical understandings and skills to solve problems. Regular work with the Standards for Mathematical Practice (the focus of this unit) reinforces the elements of *rigor* addressed in Unit 3. It is the interaction of these key advances of *focus*, *coherence*, and *rigor* within the CCR Standards, along with a clear emphasis on the Standards for Mathematical Practice, that results in a set of standards that will prepare adult learners for college and careers.

Introduce the Standards for Mathematical Practice.

Slide 3: Read through the titles of the Standards for Mathematical Practice on the slide. Then have participants work together at their tables and, using the Standards for Mathematical Practice resource, highlight the keywords in the full text for each one. (If participants received the Standards for Mathematical Practice in advance, ask them to review their notes and selections of keywords.)

Ask table leaders to facilitate discussions of participants' rationales for the selection of keywords for each Standard for Mathematical Practice. This discussion will allow participants to begin to establish a clear, and agreed on, "definition" for each of the Standards for Mathematical Practice. Remind them that these definitions should be considered preliminary. Through the Unit 4 activities, as well as in the application of the Standards for Mathematical Practice to lessons and student learning, their understanding of the Standards for Mathematical Practice should deepen and change.

Follow up on the small-group discussions with a whole-group discussion to share thoughts and insights on the meaning behind the Standards for Mathematical Practice.

Providing time at this point for whole-group discussion (and support) will help participants understand the requirements of the Standards for Mathematical Practice more clearly. For example:

- Many educators struggle with understanding mathematical modeling (MP.4). Approaches to modeling may look different across the levels. A common misconception is that any problem presented verbally requires and represents modeling. This is not necessarily true. Modeling is the process of choosing and using appropriate mathematics to analyze, understand, and make decisions based on empirical situations. Some models are very simple, but all require the student to make a choice about what kind of model to use. A real model will not come “ready-made,” with built-in supports. The variables are not labeled and the quantities, processes, and steps are not identified or provided.
- The strategic use of tools (MP.5) also is often misunderstood. This stems from the fact that tools may be used in some way in nearly all mathematics lessons. This does not mean, however, that MP.5 will be aligned to all mathematics lessons.

When a Standard for Mathematical Practice is identified as “central” to a lesson, it is needed for and closely related to the focus of the lesson. When MP.5 is identified as “central” to a lesson, the expectation is that the focus of the lesson is about how or when it is appropriate to use a certain tool. For example, a middle school lesson asks students to use a graph grid to graph an equation of a line. Even though the graph grid could technically be called a “tool,” there is no strategizing on the student’s part about whether or how to use it. On the other hand, if a lesson asks students to summarize

a bivariate data set, and the student elects to create a dot plot as a means of summarizing the data, that is an example of strategic use of the graph. In both of these examples, a graph is used as a tool. Only in the second example would the strategic reasoning of MP.5 be addressed.

Slide 4: Review the objectives of the unit with participants: 1) to recognize the Standards for Mathematical Practice as important processes and proficiencies; 2) to understand how to recognize opportunities for observing the Standards for Mathematical Practice in the requirements of both standards and lessons; and 3) to develop an understanding of how to integrate the Standards for Mathematical Practice with content in lessons as a means to enrich student learning.

Slide 5: The Standards for Mathematical Practice rest on “processes and proficiencies” with established significance in mathematics education, such as the National Council of Teachers of Mathematics and National Research Council. Tell participants that a number of surveys of employers and professors of first-year, credit-bearing mathematics courses have been conducted. The results of the surveys also indicate the importance of the processes and proficiencies described in the Standards for Mathematical Practice. They indicate that it is not enough for students to be able to perform mathematical operations and procedures. College- and career-ready students need to be able to make sense of problems and determine how mathematics can be used to solve them. They need to be able to justify their own reasoning and critique the reasoning of others. They also need to be able to model problems arising in everyday life, society, and the workplace.

Slide 6: Instruction aligned with the CCR Standards means that teachers would integrate the Standards for Mathematical Practice into level-appropriate, content-focused lessons. It is important for teachers to understand

which Standards for Mathematical Practice are central to a lesson and teach the lesson with those Standards for Mathematical Practice in mind. The goal is *not* to simply list all eight Standards for Mathematical Practice for every lesson or unit, but rather to systematically and specifically address the eight Standards for Mathematical Practice—in connection with content—across a course or level.

Begin the hands-on activity of Matching the Standards for Mathematical Practice to Content Standards.

Slides 7-8: *Distribute the participants' materials for Part One, Matching Standards for Mathematical Practice to Content Standards.* Materials include directions for Part One, the Matching Standards for Mathematical Practice to Content Standards worksheet, and the CCR Standards for Mathematical Practice. Make sure participants have the correct materials for the activity. Encourage educators needing more experience with the standards to refer to the CCR Standards document, and particularly to the Standards for Mathematical Practice.

Slides 9-10: *Provide directions to participants on how to complete the activity for Part One.* Instruct the whole group to carefully read the Level B standard provided on the Matching Standards for Mathematical Practice to Content Standards worksheet. As participants try to match the Standards for Mathematical Practice to the requirements of a specific Level B standard, they first need to imagine a lesson that would target the standard and then determine which of the Standards for Mathematical Practice are most likely to be observed during that lesson. Discuss and agree on the content and performance requirements of the standard. Be sure all instructions are clearly understood before beginning the activity. Emphasize that there are likely to be a variety of answers and rationales because there will be a variety of imagined lessons; there are no single “right” answers for this activity, but a number of well-reasoned responses.

Remind table leaders and participants that during discussion they should include how differences in the types of lessons affect their decisions regarding the relevance of the individual Standards for Mathematical Practice. One of the key objectives of this activity is for participants to begin to understand that the Standards for Mathematical Practice are most clearly observed in the context of a lesson rather than a standard. Instruct participants to work with a partner.

Here are the directions to give participants:

1. Carefully read the CCR Level B content standard on the worksheet.
2. Imagine a lesson that targets the standard. Then follow these steps:
 - Use the code X to mark Standards for Mathematical Practice that are central to the lesson you envision.
 - Use the code O to mark Standards for Mathematical Practice that support the lesson you envision.
 - Leave blank any Standards for Mathematical Practice that are not relevant to the lesson you envision.
3. Working first independently and then with a partner, evaluate the relevance of each Standard for Mathematical Practice to the requirements of the content standard listed.
4. Discuss individual decisions and rationales at your table, including how the different types of lessons imagined would affect the relevance of a particular Standard Mathematical Practice.

TIP: As differences in rationales arise, help participants understand how they may have identified different Mathematical Practices in the standard because they imagined different lessons.

Step 3: Implementation of Part Two, Enriching a Mathematics Lesson (60 minutes working in pairs or small groups)

Begin the second activity in this unit, Enriching a Mathematics Lesson.

Slide 11: *Distribute the participants' materials for Enriching a Mathematics Lesson.* Materials include Directions for Part Two, the Enriching a Mathematics Lesson worksheet, and the Math Lesson—Equivalent Fractions resource. Make sure participants have the correct materials for the activity. Ask participants to continue to use the CCR Standards for Mathematical Practice for this activity.

Slide 12: *Provide directions to participants on how to complete the activity for Part Two.* Emphasize that there are likely to be a variety of answers and rationales for this activity; there are no single “right” answers, but a number of well-reasoned responses. Review with participants the meaning of the codes X, O, and “blank.” Begin this activity working in smaller groups at tables, identifying the Standards for Mathematical Practice that are central to, or used in support of, the goals of the lesson. Here are the directions to give participants:

1. Scan the lesson and make notes about how the CCR Standards for Mathematical Practice might be observed in the activities of the lesson.
2. Use codes X and O to mark the central and supporting Standards for Mathematical Practice. Leave blank any Standards for Mathematical Practice that are not relevant to the requirements of the lesson.
3. Discuss individual decisions and rationales at your table.

Reflections: Thinking Back and Looking Forward (20 minutes)

Slide 13: After participants complete the hands-on activities, bring the whole group together. Ask them to reflect on and then discuss what they have learned. Here are some questions to encourage and inspire participants' reflection:

- What are some of the Standards for Mathematical Practice that you think are central to the lesson?
- What are some of the Standards for Mathematical Practice that you think might serve in a supporting capacity to the lesson?
- Why is it important to emphasize the Standards for Mathematical Practice at all levels of adult learning?

Point out to participants that they first looked for connections to the Standards for Mathematical Practice based solely on a standard. Then, they looked for possible evidence of the Standards for Mathematical Practice in a lesson. It should be clear that the Standards for Mathematical Practice are best “seen” in the context of the lesson and that the variation in the “imagined” lessons for Part One can produce some very different outcomes. Encourage participants to continue their exploration of the Standards for Mathematical Practice to further their understanding. There are many websites where participants can find more information about the CCR Standards for Mathematical Practice. Some examples include: InsideMathematics.org, MathEdLeadership.org (NCSM), CoreStandards.org, MathPractices.edc.org.

Slide 14: Conclude the reflections and the unit by asking participants to discuss what they have learned and consider what changes in current practice this approach to teaching

mathematics might require. Below are some questions for participants' reflections on their next steps:

- How has participating in this activity changed your thinking about the CCR Standards?
- How will you use the information and understanding you have acquired to improve your teaching practice and student learning?
- What additional training and tools would strengthen your ability to do so?